ACT Math Review

Regular Sections

Groups of Numbers

Complex Numbers

ALL numbers are complex numbers

Imaginary Numbers

⇨ Numbers that use "i"

⇨ Square roots of negative numbers

$\sqrt{-81} = 9i$

Real Numbers

If the number is not imaginary, it is real.

Groups of Numbers

Real Numbers

Includes rational and irrational numbers

Irrational Numbers

⇨ Imperfect Square Roots

$\sqrt{10}$

⇨ Pi

π

Rational Numbers

If the number is not irrational, it is rational.

$\sqrt{25}$ $\frac{1}{6}$ -8.4

Groups of Numbers

Integers

Numbers you see on a number line:

…–5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5…

Natural Numbers

⇨ Counting Numbers

1, 2, 3, 4, 5, 6, 7…

Whole Numbers

⇨ Natural Numbers and Zero

0, 1, 2, 3, 4, 5, 6, 7…

Factors of a Number

Factors are whole numbers that divide evenly into a number.

Factors of 24

Factors of 24:
1, 2, 3, 4, 6, 8, 12, 24

$1 \times 24 = 24$ $2 \times 12 = 24$
$3 \times 8 = 24$ $4 \times 6 = 24$

Factors of 30

Factors of 30:
1, 2, 3, 5, 6, 10, 15, 30

$1 \times 30 = 30$ $2 \times 15 = 30$
$3 \times 10 = 30$ $5 \times 6 = 30$

Prime Numbers

Prime numbers are whole numbers bigger than 1 that only have factors of 1 and itself.

7 is prime

Factors of 7:
1 and 7
(1×7)

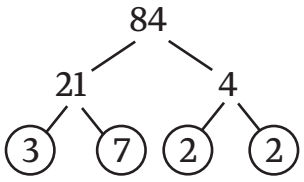
23 is prime

Factors of 23:
1 and 23
(1×23)

Other examples of primes: 2, 3, 5, 11, 13, 17

Prime Factorization

Prime Factorization is when a whole number is written as a product of prime numbers.



2	84	84 ÷ 2 = 42
2	42	42 ÷ 2 = 21
7	21	21 ÷ 7 = 3
3	3	3 ÷ 3 = 1
	1	

$84 = 2 \times 2 \times 3 \times 7 = 2^2 \times 3^1 \times 7^1$

Greatest Common Factor

The GCF is the biggest number that divides evenly into a group of numbers.

GCF of 12 and 16

Factors of 12:
1, 2, 3, *4*, 6, 12

Factors of 16:
1, 2, *4*, 8, 16

GCF = 4

GCF of 21 and 22

Factors of 21:
1, 3, 7, 21

Factors of 22:
1, 2, 11, 22

GCF = 1

Multiples of a Number

Multiples are found by skip counting.
You can multiply or add to find multiples.

Multiples of 3

Start at 3 and add by 3:
Start at *3* 3 + 3 = *6*
6 + 3 = *9* 9 + 3 = *12*

Multiply by 1, 2 and so on:
3 × 1 = *3* 3 × 2 = *6*
3 × 3 = *9* 3 × 4 = *12*

The multiples of 3 are: 3, 6, 9, 12, 15, 18, 21…

Least Common Multiple

The LCM is the smallest multiple that a group of numbers have in common.

LCM of 4 and 10

Multiples of 4:
4, 8, 12, 16, *20*…

Multiples of 10:
10, *20*, 30, 40, 50…

LCM = 20

LCM of 5 and 6

Multiples of 5:
5, 10, 15, 20, 25, *30*…

Multiples of 6:
6, 12, 18, 24, *30*…

LCM = 30

Least Common Denominator

The LCD is the smallest multiple that a group of denominators have in common.

Find the LCD of $\frac{1}{6}$ and $\frac{2}{7}$

The denominators are 6 and 7.

Multiples of 6:
6, 12, 18, 24, 30, 36, *42*…

Multiples of 7:
7, 14, 21, 28, 35, *42*…

LCD = 42

Finding Fractions of Amounts

Multiply the fraction by the amount.

3/5 of the 20 fruit in a basket are apples.
How many apples are in the basket?

Find 3/5 of 20:

$$\frac{3}{5} \times 20 = \frac{3}{5} \times \frac{20}{1} = \frac{60}{5} = \boxed{12}$$

12 apples are in the basket.

Word Problems Involving One Whole

John has read 3/8 of a book.
What fraction of the book has not been read?

$$\underset{\text{Whole Book}}{1} - \underset{\text{Read}}{\frac{3}{8}} \Rightarrow \frac{8}{8} - \frac{3}{8} = \underset{\text{Not Read}}{\boxed{\frac{5}{8}}}$$

5/8 of the book has not been read.

Finding the Halfway Point Between Fractions

Add the fractions and divide by 2.

What fraction is halfway between 3/4 and 7/8?

Add the fractions:

$$\frac{3}{4} + \frac{7}{8} = \frac{6}{8} + \frac{7}{8} = \frac{13}{8}$$

Now divide by 2:

$$\frac{13}{8} \div \frac{2}{1} = \frac{13}{8} \times \frac{1}{2} = \boxed{\frac{13}{16}}$$

Minimizing and Maximizing Fractions

The range of x is $1 \le x \le 5$. The range of y is $3 \le y \le 6$.
Find the biggest and smallest values of $\frac{x}{y}$.

Biggest Fraction:

$$\frac{\text{Big Numerator}}{\text{Small Denominator}}$$

$\frac{x}{y} \Rightarrow$ Big x / Small y $\Rightarrow \frac{5}{3}$

Smallest Fraction:

$$\frac{\text{Small Numerator}}{\text{Big Denominator}}$$

$\frac{x}{y} \Rightarrow$ Small x / Big y $\Rightarrow \frac{1}{6}$

ACT Math Review

30+ 30+

Advanced Sections

Dimensional Analysis

When converting rates, you cancel units until the units you want are left over.

Convert 9 feet per second to yards per minute

Write the rate as a fraction: Cancel feet: Cancel seconds: $\frac{9 \cdot 60}{3} = 180$

$$\frac{9\ \cancel{\text{feet}}}{1\ \cancel{\text{sec}}} \times \frac{1\ \text{yard}}{3\ \cancel{\text{feet}}} \times \frac{60\ \cancel{\text{sec}}}{1\ \text{min}} = \boxed{180 \text{ yards per min}}$$

Direct Variation

"y" varies directly with "x"

$$y = kx$$

When a table has a direct variation,

$\frac{y}{x} = k$

for all (x,y) pairs

$y = 3x$

x	y	k = 3
1	3	⇨ 3/1 = 3
2	6	⇨ 6/2 = 3
3	9	⇨ 9/3 = 3

Inverse Variation

"y" varies inversely with "x"

$$y = \frac{k}{x}$$

When a table has a inverse variation,

$xy = k$

for all (x,y) pairs

$y = \frac{18}{x}$

x	y	k = 18
1	18	⇨ 1 • 18 = 18
2	9	⇨ 2 • 9 = 18
3	6	⇨ 3 • 6 = 18

Piecewise Functions

Piecewise functions are made up of multiple functions that are turned on at different times.

$$f(x) = \begin{cases} x^2 & \text{for } x < 6 \\ x + 1 & \text{for } x \geq 6 \end{cases}$$

Find f(5)	Find f(6)	Find f(7)
$x = 5 \rightarrow x < 6$	$x = 6 \rightarrow x \geq 6$	$x = 7 \rightarrow x \geq 6$
Plug into x^2	Plug into $x + 1$	Plug into $x + 1$
$f(5) = (5)^2 = 25$	$f(6) = (6) + 1 = 7$	$f(7) = (7) + 1 = 8$

Solving Exponential Equations

Make sure the bases are the same. Then set the powers equal to each other.

SOLVE: $9^{x+3} = 81^{2x}$

$9^{x+3} = (9^2)^{2x}$ $\quad 9^2 = 81$

$9^{x+3} = 9^{4x}$

$\Downarrow$

$x + 3 = 4x$

$x = 1$

Factoring Sum and Difference of Cubes

Sum of Cubes

$$A^3 + B^3 \Rightarrow (A + B)(A^2 - AB + B^2)$$

A^3: A • A • A $\quad$ B^3: B • B • B $\quad$ S O AP

Difference of Cubes

$$A^3 - B^3 \Rightarrow (A - B)(A^2 + AB + B^2)$$

A^3: A • A • A $\quad$ B^3: B • B • B $\quad$ S O AP

The Quadratic Formula

When an equation is in the form $ax^2 + bx + c = 0$, we can find solutions using the quadratic formula.

$ax^2 + bx + c = 0$

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$3x^2 + 5x - 2 = 0$

$a = 3 \quad b = 5 \quad c = -2$

$$\frac{-5 \pm \sqrt{5^2 - 4(3)(-2)}}{2(3)}$$

$x = 1/3 \qquad x = -2$

The Discriminant

When an equation is in the form $ax^2 + bx + c = 0$, we can predict its solutions using the discriminant.

$ax^2 + bx + c = 0$

Discriminant:

$D = b^2 - 4ac$

- $D > 0$ (2 Real Solutions)
- $D = 0$ (1 Real Solution)
- $D < 0$ (No Real Solutions)

Fractional Exponents

When the exponent is a fraction, the numerator is the *power*, and the denominator is the *root*.

$x^{\frac{A}{B}}$ — A → Power, B → Root

$$x^{\frac{1}{8}} \Rightarrow \sqrt[8]{x^1} = \sqrt[8]{x}$$

$$x^{\frac{2}{5}} \Rightarrow \sqrt[5]{x^2}$$

Solving for Variables Inside of Roots

To solve for variables that are inside roots, isolate the root and use a power to cancel the root.

Solve the equation for the value of x:

$\sqrt{x} - 3 = 5$

$\sqrt{x} - 3 = 5$

$\sqrt{x} = 8$

$(\sqrt{x})^2 = (8)^2$

$x = 64$

Solving for Variables Raised to Powers

To solve for variables that are raised to powers, isolate the power and use a root to cancel the power.

Solve the equation for the value of x:

$x^3 + 4 = 12$

$x^3 + 4 = 12$

$x^3 = 8$

$\sqrt[3]{x^3} = \sqrt[3]{8}$

$x = 2$

Consecutive Integers

Consecutive Integers	Consecutive Even/Odd Integers
1, 2, 3, 4, 5, 6, 7, 8…	Even: 2, 4, 6, 8, 10… Odd: 1, 3, 5, 7, 9…

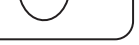
Setup:

$n \rightarrow$ 1st integer
$n + 1 \rightarrow$ 2nd integer
$n + 2 \rightarrow$ 3rd integer

Setup:

$n \rightarrow$ 1st integer
$n + 2 \rightarrow$ 2nd integer
$n + 4 \rightarrow$ 3rd integer

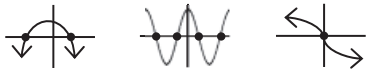
Other Graph Characteristics

Parabola $y = x^2$ 

Linear $y = mx + b$ 

Abs. Value $y = |x|$ 

The zeros of a function are located where $y = 0$ or where the graph crosses the x–axis.



Domain: All the inputs (x's) of a function.
Range: All the outputs (y's) of a function.

Comparing Functions on a Graph

Let f(x) and g(x) be functions on a graph:

$f(x) > g(x)$	$f(x) = g(x)$	$f(x) < g(x)$
It is where f(x) is *above* g(x)	It is where f(x) *crosses* g(x)	It is where f(x) is *below* g(x)